SCHEDULE AND ABSTRACTS FOR WORKSHOP AT UIC ON RANDOM MATRICES AND RANDOM POLYNOMIALS

All talks are in Room 241 of UIC's Academic and Residential Complex (ARC).

Schedule

Day 1: May 15

9-9:30: Welcome and Coffee
9:30-10:30: Konstantin Tikhomirov
10:45-11:20: Andrew Campbell

11:20-1PM: Lunch

1-1:30: Coffee/refreshments

1:30-2:30 Galyna Livshyts
2:45-3:20 Marcelo Campos

Day 2: May 16

9-10: Nick Cook
10:15-11: Open Problem Session

11-12:30PM: Lunch

12:30-1:30: Oanh Nguyen
1:45-2:20: Max Xu

Titles and Abstracts

Konstantin Tikhomirov

On the probability that the convex hull of random points contains the origin

We consider the problem of computing the probability that the convex hull of random vectors with i.i.d symmetrically distributed components contains the origin. Applying tools from non-asymptotic random matrix theory, we show that the probability deviates from Wendel's formula by an exponentially small additive term. We will further discuss the result in the context of average-case analysis of linear programs.

Andrew Campbell

Derivatives of random polynomials and sums/corners of random matrices

Recently, a surprising connection has been established between repeated differentiation of random polynomials and sums of random matrices. For polynomials with real roots and self-adjoint matrices, the evolution of the empirical root/spectral measures under these processes can be described in terms of the free additive convolution. The free additive convolution, which defines the asymptotic spectrum of sums of random self-adjoint matrices, can also be characterized in terms of corners of a single random matrix. We will discuss progress on extending this connection to polynomials with complex roots and non-self-adjoint matrices. Specifically, we will see a free probabilistic interpretation for repeated differentiation of random polynomials with independent coefficients, with a convolution coming from sums of single ring matrices. Alternatively, as with the real case, this convolution can be defined in terms of corners of a one single ring matrix, simplifying key computations. Interpreting repeated differentiation as a convolution naturally leads to consideration of stable laws and a central limit theorem. We will discuss these stable laws and other consequences of extending this connection to the complex setting. This is joint work with Sean O'Rourke and David Renfrew.

Galyna Livshyts

Invertibility of inhomogenuous heavy-tailed matrices

We will show the sharp estimate on the behavior of the smallest singular value of random matrices under very general assumptions. One of the key steps in the proof is a result about the efficient discretization of the unit sphere in an n-dimensional euclidean space. The proof of the result will be outlined. Partially based on the joint work with Tikhomirov and Vershynin.

Marcelo Campos

The least singular value of a random symmetric matrix

Let A be a $n \times n$ symmetric matrix with $(A_{i,j})_{i \leq j}$ independent and identically distributed according to a subgaussian distribution. In this talk I will present a recent result that shows

$$\mathbb{P}(\sigma_{\min}(A) \leq \varepsilon/\sqrt{n}) \leq C\varepsilon + e^{-cn},$$

where $\sigma_{\min}(A)$ denotes the least singular value of A and the constants $C, c > 0$ depend only on the distribution of the entries of A. This is joint work with Matthew Jenssen, Marcus Michelen and Julian Sahasrabudhe.

Nick Cook

Large deviations for the largest eigenvalue of sub-Gaussian Wigner matrices

Until recently, large deviation principles (LDP) for spectral statistics of random matrices were largely limited to the Gaussian ensembles. In a breakthrough work, Guionnet and Husson established the LDP for the largest eigenvalue of Wigner matrices having "sharp sub-Gaussian" entries, which includes Rademacher variables. Moreover, they showed the rate function for GOE matrices is universal for this class. I will discuss new results for the general sub-Gaussian case, where localization phenomena can give rise to non-universal rate functions. This is joint work with Raphael Ducatez and Alice Guionnet.

Oanh Nguyen

Hole radii for the Kac polynomials

The Kac polynomial is one of the most studied models of random polynomials. It has the form

$$f_n(x) = \sum_{i=0}^{n} \xi_i x^i.$$

It is known that the empirical measure of the roots converges to the uniform measure on the unit disk. On the other hand, at any point on the unit disk, there is a hole in which there are no roots, with high probability. In a beautiful work, Michelen showed that the holes at ± 1 are of order $1/n$. We show that in fact, all the hole radii are of the same order. Joint work with Hoi Nguyen.

Max Xu

Recent progress in random multiplicative functions

Random multiplicative functions are probabilistic models for arithmetic functions that number theorists care about. It also has strong connections to problems in additive combinatorics and becomes an active area. In this talk, I will survey the subject and describe some recent progress as well as some open problems.